Dynamical screening and radiative parton energy loss in a quark-gluon plasma

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Abstract

Dynamical screening in the magnetic part of the one-gluon exchange interaction is included in the study of radiative energy loss of a fast parton propagating inside a quark-gluon plasma. As a result the final radiative energy loss is about twice as large as when only the electric part of one-gluon exchange interaction is considered. A non-perturbative magnetic screening mass is also used in the estimate of the mean-free-path of parton scattering in a hot QCD matter. © 2000 Elsevier Science B.V. All rights reserved.

Radiative energy loss of a fast parton inside a hot QCD matter has been proposed as a good probe of the medium and should lead to observable consequences such as jet quenching in high-energy heavy-ion collisions [1–4]. Theoretical estimate of the radiative energy loss suffered by a fast parton in a hot QCD medium has attracted a lot of interests because it helps us to understand the dependence of the energy loss on the properties of the medium and in particular the difference between parton energy loss in a cold nuclear medium and a hot quark-gluon plasma.

Radiative energy loss has been estimated in various approaches, from uncertainty principle analysis [5] to calculation of induced radiation in a multiple scattering model [6]. A very interesting feature of the radiative energy loss found by a recent study in Ref. [7], referred to as BDMPS in this paper, is that the energy loss depends quadratically on the distance that the parton travels through. BDMPS demonstrated that such a nonlinear dependence arises from the non-abelian gluon rescattering in the medium. Most of these studies used the screened static-potential model for multiple scattering in a hot medium as proposed by Gyulassy and Wang (GW) [6]. In the GW model for multiple scattering, the interaction suffered by the propagating parton is assumed to be by a static potential with Debye screening. Such a screened static potential model gives finite cross section and average transverse momentum broadening. Even though BDMPS and Zakharov [8,9] later on generalized the study to other models of parton scattering, the problem of the magnetic part of one-gluon exchange interaction in a medium and its

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effect on the radiative energy loss remains unexplored.

In this paper we will study the radiative energy loss of a fast parton inside a quark-gluon plasma including both the electric and magnetic part of the strong interaction. The magnetic part of the one-gluon exchange interaction is not screened perturbatively in the static limit in a hot QCD plasma. One therefore has to introduce a non-perturbative magnetic screening mass μ_{mag} in order to calculate the parton scattering cross section or the mean-free-path of a propagating parton similar to the calculation of the gluon damping rate [10]. For the calculation of some transport quantities, like the average momentum transfer per interaction, the dynamical screening provided by the imaginary part of the self-energy in the magnetic interaction is enough to regulate the infrared behavior of the magnetic interaction and gives finite results. In both cases, correlation scales provided by the static and dynamics magnetic screening are somewhat different from the static electric screening. They should have significant effect on radiative parton energy loss in a quark-gluon plasma.

In studies [6,7] of the parton energy loss in a medium, a static model of parton scattering is normally used which was shown [6] to be equivalent to the non-static case in a physical gauge ($A^+ = 0$) as far as the radiation amplitudes are concerned. The final result of radiative parton energy loss is very general. It only depends on the averaged transverse momentum broadening. However, the estimate of the transverse momentum broadening depends on the modeling of the parton interaction. We will show that both the electric and magnetic part of the interaction contribute to the transverse momentum broadening. For estimate of the mean-free-path of parton scattering, the magnetic interaction actually dominates because of the lack of perturbative static screening.

According to BDMPS [7], the radiative energy loss of a fast parton inside a medium with finite size L is

$$\frac{dE}{dz} = \frac{\alpha_s N_c}{4} \langle p^2_{\perp W} \rangle , \qquad (1)$$

for any model of multiple parton scattering, where $\langle p^2_{\perp W} \rangle$ is the total accumulated momentum broad-

ening during the parton's propagation inside the medium which grows linearly with the media length L, i.e., $\langle p^2_{\perp W} \rangle = L d\langle p^2_{\perp} \rangle / dL$. The momentum broadening per unit distance is

$$\frac{d\langle p^2_{\perp} \rangle}{dL} = \rho \int_0^{\mu^2/B^2} dq^2 q^2 \frac{d\sigma}{dq^2} , \qquad (2)$$

where ρ is the media parton density, $B = \lambda/L$ with λ being the mean-free-path of the propagating parton and μ is the typical momentum transfer in a parton scattering which is the Debye screening mass μ_D in the GW model of multiple scattering. In a hot quark-gluon plasma, one should include both the electric and magnetic interaction of one-gluon exchange. One should also replace Eq. (2) with its thermal averaged value,

$$\frac{d\langle p^2_{\perp} \rangle_a}{dL} = \sum_b \nu_b \int \frac{d^3 p_b}{(2\pi)^3} f(p_b)\left(1 \pm f(p_c)\right)$$

$$\times \frac{d^3 p_c}{(2\pi)^3} \frac{d^3 p_d}{(2\pi)^3} q^2 |\mathcal{M}_{ab}|^2$$

$$\times (2\pi)^4 \delta^4(p + p_b - p_c - p_d) , \qquad (3)$$

where we use the index a to denote the flavor of the fast parton and $f(p)$ is the Bose-Einstein f_{BS} (Fermi-Dirac f_{FD}) distribution for the thermal gluons (quarks) in the medium. We will only consider the elastic channels that are dominant at small angles. The statistical factor ν_2 is $2(N_c^2 - 1)$ for gluons and $4N_c n_f$ for n_f flavors of quarks. In this paper we will assume $n_f = 2$. We neglect the quantum statistical effect for the fast partons. We denote the energy and momentum transfer of the parton scattering by ω and q, respectively. The above integral is dominated by contributions from small angle scattering. In this small-angle approximation, i.e., $\omega, q \ll E, E_b$, energy-momentum conservation leads to

$$p_c = p + q , \quad p_d = p_b - q ,$$

$$E_c = E + \omega , \quad E_d = E_b - \omega ,$$

$$\omega \approx v \cdot q \approx v_b \cdot q , \qquad (4)$$

where $\mathbf{v} = \mathbf{p}/E$ and $\mathbf{v}_b = \mathbf{p}_b/E_b$. In the small-angle scattering limit, the effective matrix element for parton scattering is [10],

$$\mathcal{M}_{ab} \approx g^4 C_{ab} \left[\frac{1}{q^2 + \mu_D^2 \pi_L(x)} \right.$$

$$\left. - \frac{(1-x^2)\cos\phi}{q^2(1-x^2) + \mu_D^2 \pi_T(x)} \right], \qquad (5)$$

where $\cos\phi = (\mathbf{v} \times \mathbf{q}) \cdot (\mathbf{v}_b \times \mathbf{q})/q^2$, $x = \omega/q$ and $\mu_D^2 = g^2(N_c + n_f/2)T^2/3$ is the Debye screening mass in thermal QCD medium with temperature T. The color factors for parton scattering are $C_{qq} = (N_c^2 - 1)/4N_c^2 = 2/9$, $C_{qg} = 1/2$ and $C_{gg} = N_c^2/(1 - N_c^2) = 9/8$. We use an effective gluon propagator to include the resummation of an infinite number of loop corrections [11]. The scaled self-energies in the effective propagator in the long-wavelength limit are given by [12],

$$\pi_L(x) = 1 - \frac{x}{2}\ln\left(\frac{1+x}{1-x}\right) + i\frac{\pi}{2}x, \qquad (6)$$

$$\pi_T(x) = \frac{x^2}{2} + \frac{x}{4}(1-x^2)\ln\left(\frac{1+x}{1-x}\right)$$

$$- i\frac{\pi}{4}x(1-x^2). \qquad (7)$$

In Eq. (3), the integration over p_c and p_d can be rewritten as

$$(2\pi)^4 \int \frac{d^3p_c}{(2\pi)^3} \frac{d^3p_d}{(2\pi)^3} \delta^4(p + p_b - p_c - p_d)$$

$$= \frac{1}{(2\pi)^2} \int d^3q \int_{-q}^{q} d\omega \, \delta(\omega - \mathbf{v} \cdot \mathbf{q})$$

$$\times \delta(\omega - \mathbf{v}_b \cdot \mathbf{q}). \qquad (8)$$

The two δ-functions will fix the angular integrals of p_b and q. With approximation $f(p_c) \approx f(p_b)$ and the integrals

$$\int dp\, p^2 f_{\mathrm{BS}}(p)(1 + f_{\mathrm{BS}}(p)) = T^3\pi^2/3,$$

$$\int dp\, p^2 f_{\mathrm{FD}}(p)(1 - f_{\mathrm{FD}}(p)) = T^3\pi^2/6, \qquad (9)$$

we obtain the averaged momentum transfer per unit distance in Eq. (3) as

$$\frac{d\langle p_\perp^2 \rangle_a}{dL} = \frac{g^4}{2\pi} C_a T^3 \int_{-1}^{1} dx \int_{0}^{q_{\max}^2/B^2} \frac{dq^2 q^2}{\mu_D^4}$$

$$\times \left\{ \frac{1}{|q^2/\mu_D^2 + \pi_L(x)|^2} \right.$$

$$\left. + \frac{1}{2} \frac{(1-x^2)^2}{|(1-x^2)q^2/\mu_D^2 + \pi_T(x)|^2} \right\}, \qquad (10)$$

where $C_q = 4/9$ and $C_g = 1$. The integration over p_b provides a cut-off for q integration at $q_{\max}^2 = 3ET/2$. The integral in Eq. (10) should be a function of $q_{\max}^2/\mu_D^2 B^2$ only.

From Eq. (6) and (7), we can see that with Debye screening the longitudinal contribution to the above integral is finite. However, the real part of the transverse self-energy vanishes quadratically in the static limit ($x \to 0$). Without the imaginary part this would have caused a quadratical divergency in the transverse propagator. Fortunately, imaginary part provides Landau damping to parton interactions in a thermal medium and reduce the divergency to a logarithmic singularity in the static limit. When weighted with the momentum transfer q^2, the transverse contribution to the integral is then finite. A fit to the numerical evaluation of the integral gives,

$$\mathcal{I}_L = 0.92\ln\frac{q_{\max}^2}{\mu_D^2 B^2}, \qquad \mathcal{I}_T = 0.5\ln\frac{q_{\max}^2}{\mu_D^2 B^2}. \qquad (11)$$

We can see that the contribution from the magnetic interaction is as big as the electric one. The final momentum broadening per unit distance and the radiative parton energy loss are then

$$\frac{d\langle p_\perp^2 \rangle_a}{dL} = 8\pi C_a T^3 \alpha_s^2 1.42\ln\frac{3ETL^2}{2\mu_D^2 \lambda_a^2},$$

$$\frac{dE_a}{dz} = \frac{N_c \alpha_s}{4} L \frac{d\langle p_\perp^2 \rangle_a}{dL}. \qquad (12)$$

In order to complete the estimate of the radiative energy loss, we now have to estimate the mean-free-path of parton scattering in the same framework. Similarly including both the electric and magnetic part of interaction, one has

$$\lambda_a^{-1} \equiv \langle \rho\sigma \rangle_a = \frac{g^4}{2\pi} C_a \frac{T^3}{\mu_D^2} \int_{-1}^{1} dx \int_0^{q_{max}^2} \frac{dq^2}{\mu_D^2}$$

$$\times \left\{ \frac{1}{|q^2/\mu_D^2 + \pi_L(x)|^2} \right.$$

$$\left. + \frac{1}{2} \frac{(1-x^2)^2}{|(1-x^2)q^2/\mu_D^2 + \mu_{mag}^2 + \pi_T(x)|^2} \right\}.$$
$$(13)$$

Unlike in Eq. (10) for the averaged momentum transfer, the logarithmic singularity in the magnetic part of the gluon propagator can only be regularized by introducing a non-perturbative magnetic screening mass $\mu_{mag} \sim g^2 T$ much like in the calculation of the damping rate of a fast parton [10,11]. In the weak coupling limit, the dominant contribution in the magnetic interaction comes from $\mu_{mag} \lesssim q \lesssim \mu_D$ and is independent of the cut-off $q_{max} \gg \mu_D$. Numerically one can carry out the above integration and find the integral as

$$\mathscr{I}_\lambda = 2\left(\ln\frac{\mu_D^2}{m_{mag}^2} - 1.0 + 2.0\frac{\mu_{mag}^2}{q_D^2} - 0.32\frac{\mu_D^2}{q_{max}^2} \right)$$

$$+ 2.2\frac{q_{max}^2}{q_{max}^2 + \mu_D^2}$$

$$\approx 2\left(\ln\frac{\mu_D^2}{m_{mag}^2} - 0.1 \right) + \mathscr{O}(g^2), \qquad (14)$$

where the first term is the magnetic and the second term is the electric contribution. Using the estimate of $\mu_{mag} \approx 0.255\sqrt{N_c/2}\, g^2 T$ from Ref. [13], we have

$$\lambda_a^{-1} = 3TC_a\alpha_s \ln\frac{1}{\alpha_s}, \qquad (15)$$

with the dominant contribution from the magnetic interaction. This a perturbative result with a non-perturbative magnetic screening mass as a cut-off in the

magnetic interaction. Such a cut-off is not necessary if one can find a way to eliminated the infrared divergence via non-perturbative method [14]. In the framework of the effective theory, it was recently pointed out [15,16] that the infrared divergence in the magnetic interaction should instead be regulated by a scale $\mu \sim g^2 T \ln(1/g)$ separating hard and semi-hard processes from the non-perturbative processes at ultra-soft momenta ($< g^2 T < \mu$). The μ dependence of the final result should be canceled when the non-perturbative contribution is matched to the perturbative one. Without considering non-perturbative contribution which is outside the scope of this paper, our estimate of the mean-free-path in Eq. (15) is only good to the order $\alpha_s\ln(1/\alpha_s)$ no matter whether we use magnetic mass μ_{mag} or the intermediate scale μ as the infrared cut-off.

In the weak coupling limit, the magnetic contribution significantly changes the mean-free-path of a propagating parton. However, for a practical value of $\alpha_s = 1/3$, the net result is about the same as when only the electric interaction with Debye screening is considered. Substitute the above mean-free-path into Eq. (12), we have the parton energy loss in a quark-gluon plasma,

$$\frac{dE_a}{dz} = 4\pi N_c \alpha_s^3 T^3 L C_a$$

$$\times 1.42\ln\left[\frac{9}{2} C_a L \ln\frac{1}{\alpha_s} \sqrt{2\pi\alpha_s ET} \right]. \qquad (16)$$

For a fast quark with $E = 250$ GeV traveling through a hot quark-gluon plasma with $T = 250$ MeV and $L \simeq 10$ fm we have

$$\frac{d\langle p_T^2 \rangle_a}{dL} \approx 1.9\,\text{GeV}^2/\text{fm},$$

$$\frac{dE}{dz} \approx 24\,\text{GeV}/\text{fm}\left(\frac{L}{10\,\text{fm}} \right). \qquad (17)$$

This is about 4 times larger than the original estimate by BDMPS [7,9]. It is in part due to the contribution from the magnetic interaction that was not considered before and in part due to the different estimate of the mean-free-path in this paper which depends on the non-perturbative magnetic screening mass μ_{mag}. For a smaller value of $\alpha_s = 1/10$ that is more

relevant for the week coupling limit in our estimate, we have

$$\frac{d\langle p_T^2 \rangle_a}{dL} \approx 0.18\,\mathrm{GeV}^2/\mathrm{fm}\,,$$

$$\frac{dE}{dz} \approx 0.68\,\mathrm{GeV}/\mathrm{fm}\left(\frac{L}{10\,\mathrm{fm}}\right). \qquad (18)$$

To conclude, we have considered both the electric and magnetic part of one-gluon exchange interaction in the estimate of radiative energy loss by a fast parton propagating in a hot QCD plasma. We used the results by BDMPS [7] for the energy loss which is proportional to the averaged momentum transfer per unit distance. The imaginary part of the magnetic self-energy which is responsible for Landau damping regularizes the collisional integral in the calculation and gives a finite averaged momentum transfer. We found the contribution from the magnetic interaction is as big as the electric interaction. In the estimate of the mean-free-path, we have to introduce a magnetic screening mass which gives an additional logarithmic dependence on the strong coupling constant $\ln 1/\alpha_s$. The final radiative energy loss has a cubic dependence on the coupling constant. Because of our treatment of the magnetic interaction, our final numerical estimate of radiative energy loss is about 4 times larger as the original estimate by BDMPS.

Acknowledgements

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